

# Spacetime Singularities and Cosmic Censorship

Pankaj S. Joshi

*Tata Institute for Fundamental Research*

*Homi Bhabha Road, Mumbai 400005, India*

We present here a brief review and discussion on recent developments in the theory of spacetime singularities. After mentioning some key motivations on the main ideas and concepts involved, we take the approach that the singularities will be eventually resolved by the quantum gravity effects. Some consequences are indicated when such singularities are visible to far away observers in the universe.

## I. INTRODUCTION

Physical phenomena in the universe take place in the arena of space, and evolve in time. The known laws of physics describe and govern these happenings that occur in nature. But then what are space and time, what are their interconnections if any, and how the space and time themselves originate, or whether each of these is actually infinite and endless? These are some of the most profound questions that have exercised greatest of minds in science and philosophy over past many centuries.

The best possible scientific theory that we have today, that governs the universe of space and time is the general theory of relativity. In relativity, the space and time are no longer separate and independent of each other, but they are intertwined with each other and the actual physical measurements of these quantities are always mutually related. While general relativity was originally developed to describe the force of gravitation, it has provided us with some of the most intriguing insights on the nature of space and time, their inter-connections, and the possible origins of the time and space itself. General relativity suggests not to view gravity as a ‘force’ in the usual sense, but describes it as the curvature and geometry of the ‘space-time continuum’, which is our universe.

General relativity describes the interplay of the space-time curvature, and the matter within it that generates such a curvature, just as a metal ball placed on a rubber sheet curves it. It is a classical theory that governs the universe in its large scale structure. Given any physical system governed mainly by gravity, such as a large collection of galaxies, or a massive star that is close to the end of its evolution having burnt all its nuclear fuel, the Einstein equations govern the future evolution of such a system in time. Thus we can ask the questions such as how the universe of

galaxies that is continuously expanding will evolve in future, and whether it will continue to expand or will it start shrinking at some point in time in future. Or one could ask, what will be the final end point of evolution of a massive star that has started contracting under the force of its own gravity when its internal fuel is exhausted.

One of the most remarkable predictions of general relativity, developed during the 1960s and early 1970s has been that, dynamical evolution of matter fields in a space-time generically produces a space-time singularity. Such a singularity is where the densities, space-time curvatures and all other physical quantities blow up and grow arbitrarily large, and thus all known physical laws no longer hold there. In that sense, the singularity is the end (or beginning) of the space and time themselves.

The singularity theorems developed by Roger Penrose, Stephan Hawking and Robert Geroch show that the evolution of matter fields in a spacetime generically yields such a singularity, provided reasonable physical conditions are satisfied such as the causality ensuring that you do not return to your own past, a suitable energy condition ensuring the positivity of energy density, and formation of what are called ‘trapped surfaces’ in a space-time that indicate and characterize that the gravitational field is sufficiently strong. The space-time singularities develop in cosmology, where they signal the beginning of time, and in gravitational collapse of massive stars, which is an issue of great interest in gravitation physics today that has been investigated in much detail in recent years in the Einstein theory.

We outline here some aspects of space-time singularities that occur in cosmology and in gravitational collapse. The singularity theorems predicting the occurrence of singularities allow the singularities of gravitational collapse to be either visible to external observers or covered by an event horizon of gravity. Some consequences of this fact are indicated. The role of space-time singularities as an inevitable feature of Einstein’s theory of gravity has become clear now as signalling the situations where the gravitational field becomes ultra-strong and grows without any upper bound. Close to the singularity is the regime of strong gravity fields, where general relativity comes into its own to imply most interesting physical consequences.

## II. THE OCCURRENCE OF SINGULARITIES

We discuss now the occurrence of space-time singularities in some detail within a general space-time framework. The basic ideas involved in the singularity theorems are indicated, and what these theorems do *not* imply is pointed out.

We observe the universe today to the very far depths in space and time through the telescopes that observe the objects which are billions of light years away. One could look deep into space, let us say in diametrically opposite directions. Then the regions with extremely distant galaxies are seen in each of these directions. It is most interesting to observe that these regions have actually quite similar properties in terms of their appearance and there is a homogeneity seen in the spatial distribution of the far away galaxies. The universe also looks similar in different directions, thus exhibiting an isotropy. These regions are, however, so far away from each other that they have had no time to interact mutually. That is because, general relativity equations imply that a homogeneous and isotropic universe had a finite age in the past when the energy density of matter is positive. The age of the universe since such a big bang that indicated the origin of both space and time has not been actually large enough for any such interactions to have taken place in the past. Thus, within the big bang framework of cosmology, a very relevant question arises: How come these regions have such similar properties? This is one of the major puzzles of modern cosmology today.

This observed homogeneity and isotropy of the universe at large enough scales can be modelled by the so called Friedmann-Robertson-Walker geometry. The metric describing the geometry of the corresponding space-time universe is given by

$$ds^2 = -dt^2 + R^2(t) \left[ \frac{dr^2}{(1 - kr^2)} + r^2 d\Omega^2 \right]. \quad (1)$$

Here  $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$  is the metric on a two dimensional sphere and the universe is assumed to be spherically symmetric here. The additional assumption here is that the matter content of the space-time is homogeneous and isotropic, to represent these observed features of the universe. The scale factor  $R(t)$  increases in time so as to model the observed expansion of the universe where galaxies recede from each other in space. Thus, the matter density is the same everywhere in the universe at any given epoch of time, and also the visual appearance of universe looks the same in all directions.

The galaxies here are represented by point-like objects which form ‘dust particles’ of this universe, which is the matter content of the space-time. Combining these geometrical features with the Einstein equations and solving the same, one is led to the Friedmann solution yielding a description of the dynamical evolution of the universe and the matter within it. The picture obtained from such an evolution implies that the universe must have had a beginning at a finite time in the past. This is the epoch of the so called big bang singularity. The matter density as well as the curvatures of spacetime diverge in the limit of approaching this cosmological singularity. This

is an epoch where all non-spacelike geodesics that represent the trajectories of the photons and the material particles come to an end and these are ‘incomplete’ at a point in the past where the space-time comes to an end.

A similar occurrence and formation of a space-time singularity takes place when a massive star collapses freely under the force of its own gravity when it has exhausted its internal fuel which made it shine earlier. If the mass of the star is small enough, it can stabilize as a white dwarf or a neutron star at the end of its life cycle, which would then be the natural endstate of its evolution. However, in case the mass is much larger, say of the order of tens of solar masses, a continual gravitational collapse of the star is inevitable once there are no internal pressures left to sustain the star. This is because no known forces can then stabilize such a star. Such a scenario was considered and modelled using general relativity by R. Oppenheimer and H. Snyder in 1939, and by B. Datt in 1938, when they considered a collapsing spherical cloud of dust. Again, according to the equations of general relativity, a space-time singularity of infinite density and curvature forms at the center of the collapsing cloud. We shall discuss gravitational collapse in some more detail in the next section.

Such space-time singularities were discovered in the context of specific models of universe or of a collapsing massive star, such as those discussed above, and after their discovery these were debated extensively by the gravitation theorists in the 1940s and 1950s. An important key question that was persistently asked at this juncture in this connection was the following: Why should these models be taken seriously at all, when they were so special because they assumed so many symmetries of space-time? As a result, perhaps such a space-time singularity was arising due to these special symmetry assumptions, and may be it could occur in such special circumstances only as described and assumed by these models, but possibly it would not actually develop in the actual physical reality which is our universe. In other words, these singularities could be just some isolated examples occurring in some special models and manifestation of the symmetry assumptions made. After all, the Einstein equations

$$R_{ab} - \frac{1}{2}Rg_{ab} = 8\pi T_{ab}, \quad (2)$$

governing the ever present force of gravity, are a complex system of second-order, non-linear, partial differential equations, which admit an infinite space of solutions. The models discussed above are only special cases and isolated examples in this full space of solutions.

Therefore, the main issue was the absence of any general enough proof that such space-time singularities would always occur in a general enough gravitational collapse depicting actual physical

systems when a massive star dies, or in a generic enough cosmological scenario. In fact, there was a widespread belief in the 1940s and 1950s that such singularities would be simply removed and go away both from stellar collapse and from the cosmological considerations of the universe (which are two very important physical situations), once assumptions such as the dust form of matter, the spherical symmetry of the model, and such others were relaxed and when more general solutions to the Einstein equations were found and considered.

This is where the work by A. K. Raychaudhuri in 1955, on the gravitational focusing of matter and light in a space-time universe became relevant. This was used by R. Penrose, S. W. Hawking, and R. Geroch, who analyzed the causal structure and global properties of a fully general spacetime, and who then combined it with the considerations on gravitational focussing effect as developed by Raychaudhuri. The culmination of these efforts was the singularity theorems in general relativity, which showed that space-time singularities, such as those depicted in the examples of gravitational collapse and cosmology we discussed above, in fact manifested themselves in a rather large class of space-time universes under quite general physical conditions.

The Raychaudhuri equation played a central role in the analysis of space-time singularities in general relativity. Prior to the use of this equation to analyse collapsing and cosmological situations for the occurrence of singularities [1] [4], most works on related issues had considered only rather special cases with many symmetry conditions assumed on the underlying spacetime. But with the help of this equation these aspects could be discussed within the framework of a general spacetime without any symmetry conditions. This was in terms of the overall behaviour of the congruences of trajectories of material particles and photons propagating and evolving dynamically. This analysis of the congruences of non-spacelike curves which represent either material particles or light rays, showed how gravitational focusing took place in the universe giving rise to what are called caustics where nearby trajectories intersect due to gravitational focussing.

Before general singularity theorems could be constructed, however, another important mathematical input was needed in addition to the Raychaudhuri equation. This was the analysis of the causality structure and general global properties of a spacetime manifold. This particular development took place mainly in the late 1960s (for a detailed discussion, see e.g. [2]). The singularity theorems then combined these two important features, namely the gravitational focusing effects due to matter and energy content of the space-time and the causal structure constraints which followed from the global spacetime properties, to obtain the existence of space-time singularities in the form of geodesic incompleteness in the space-time.

As we discussed earlier, the main question here was that of genericity of the space-time singu-

larities, either in cosmology or in collapse situations. The singularity theorems, while proving the existence of singularities in a generic manner, by themselves provide no information either on the structure and properties of such singularities, or on the growth of curvature and densities in their vicinity.

There are several singularity theorems available which establish the non-spacelike geodesic incompleteness for a spacetime under different sets of physical conditions. Each of these may be more relevant to one or the other specific physical situation, and may be applicable to different physical systems such as stellar collapse or the universe as a whole. However, the most general of these is the Hawking-Penrose theorem, which is applicable to both the collapse situation and the cosmological scenario. To outline briefly the basic idea and the chain of logic behind the same, firstly, using causal structure analysis it is shown that between certain pairs of events in space-time there must exist timelike geodesics curves of maximal length. However, both from causal structure analysis and from the global properties of a general space-time manifold (which is assumed to satisfy a specific energy condition), it follows that a causal geodesic curve, which is complete in regard to both the future and past, must contain caustics where nearby null or timelike geodesics must intersect. One is then led to a contradiction, because the maximal geodesic curves mentioned above are not allowed to contain any such conjugate points, the existence of which would be against their maximality. Thus the space-time itself must have non-spacelike geodesic incompleteness.

Such theorems do assume some physical reasonability conditions. First such condition assumed on the space-time is an energy condition. All classical fields have been observed to satisfy a suitable positivity of energy density requirement, and therefore this may be considered to be quite reasonable. The second condition is a statement that all non-spacelike trajectories do encounter some non-zero matter or stress-energy density somewhere during their entire path. This is called the genericity condition. The third is a global causality requirement to the effect that there are no closed timelike curves in the spacetime. Finally, there is a condition that relates to either a gravitational collapse situation or to gravitational focussing within a cosmological framework, which considers how the congruences of non-spacelike curves in a space-time expand or converge. If these conditions are satisfied then the theorem goes through, proving the existence of space-time singularities within a general space-time scenario without special symmetry conditions.

### III. GRAVITATIONAL COLLAPSE

Another important physical situation where space-time singularities do occur is the gravitational collapse of a massive star. In fact, it was pointed out by S. Chandrasekhar in 1935 that, "...the life history of a star of small mass must be essentially different from that of a star of large mass... A small mass star passes into White-dwarf stage... A star of large mass *cannot* pass into this stage and one is left speculating on other possibilities."

While we have pointed out above that space-time singularities must develop in gravitational collapse, the most important question that this situation gives rise to is: *What is the final fate of a massive star when it undergoes a continual gravitational collapse?* This has been one of the most important key problems in astronomy and astrophysics for past many decades. If the star is sufficiently massive, beyond the white dwarf or neutron star mass limits, then a continued gravitational collapse must ensue when the star has exhausted its nuclear fuel.

What are then the *possible end states* of such a *continued gravitational collapse* is the issue to be resolved. To answer this question, one must study dynamical collapse scenarios within the framework of a gravitation theory such as the Einstein theory. We now outline some recent developments in past couple of decades in this area on the final state of a gravitationally collapsing massive matter cloud. We point out how the black hole and naked singularity end states arise naturally and generically as spherical collapse final states. We see that it is the geometry of trapped surfaces in the space-time that governs this phenomena.

It was conjectured by Penrose in 1969, that the ultra-dense regions forming in gravitational collapse, that is the space-time singularities where the physical quantities such as densities and curvatures are having extreme values, must be hidden within the event horizon of gravity. That is, the collapse must end in a black hole. This is called the 'cosmic censorship conjecture'. There is, however, no proof or any suitable mathematical formulation available for the same as of today despite many attempts.

If the gravitational collapse always produces a black hole, then that provides a very strong foundation for the theory as well as the astrophysical applications of black holes. On the other hand, if collapse produces visible ultra-strong gravity regions, or naked singularities, then the physical processes in these super-strong gravity regions can propagate, in principle, to external observers in the universe, thus giving rise to very interesting physical consequences.

Under the situation, very many researchers have made extensive studies of various dynamical collapse models, mainly spherically symmetric, over past couple of decades, to investigate the final

outcome of a continual gravitational collapse. When no proof, or even a suitable mathematical formulation of censorship conjecture is available, it is only such studies that can throw light on this issue. The generic conclusion that follows is: Either a *black hole* or a *naked singularity* develops as end product of collapse, depending on the initial data for the collapsing matter cloud (for example, the initial density, pressures, and velocity profiles for the collapsing shells of matter), from which the collapse develops, and the nature of dynamical evolutions as permitted by Einstein equations.

While extensive study is made of astrophysics of black holes, for the visible singularities we may still want to inquire into questions such as: Are naked singularities of gravitational collapse generic, or What are the physical factors that cause a naked singularity, rather than a black hole forming as collapse end state. That is, one may wish to understand in a better way the naked singularity formation in gravitational collapse. Basically, it turns out that the black hole or naked singularity phases of collapse are determined by the geometry of the trapped surfaces that develop as the collapse evolves.

What governs the geometry of the trapped surfaces, or the formation or otherwise of the naked singularities in the spacetime? We can ask in other words, what is it that causes the naked singularity to develop rather than a black hole as collapse final state? It turns out that physical agencies such as inhomogeneities in matter profiles play an important role to distort the trapped surface geometry to delay the trapped surface formation during the collapse, thus giving rise to a naked singularity.

When the collapsing dust matter is homogeneous, the final outcome of collapse is a black hole and the singularity is hidden within the horizon. But if the collapsing cloud has a density higher at the center, then the trapped surfaces are delayed and the outcome is a naked singularity from which the light or matter particles can escape away (see Fig.1 and Fig.2).

This, in a way, provides the physical understanding of the phenomena of black hole and visible singularities occurring as end states for gravitational collapse.

What would be the outcome when the collapse is non-spherical? There are some examples which indicate the outcome to be somewhat similar in nature, but the evidence in this case is limited so far. The main difficulty is the complexity of the Einstein system of differential equations. It will be necessary to understand non-spherical collapse better before we can decide on the genericity aspect of the visible singularities.

There are several quite interesting questions which are under active investigation at the moment. For example, could naked singularities generate bursts of gravity waves? What kind of quantum effects will take place near a visible singularity? Many of these issues would have interesting physical



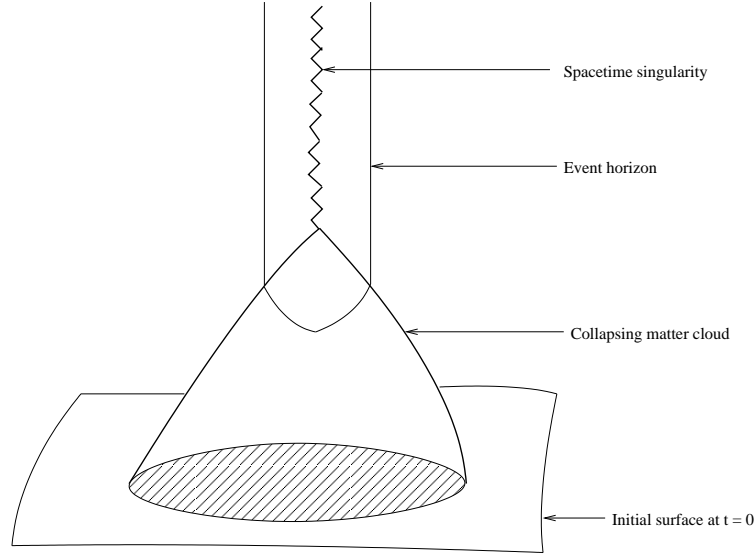


FIG. 1: The space-time singularity developing in collapse is hidden within an event horizon in the case of gravitational collapse of a homogeneous density dust cloud.

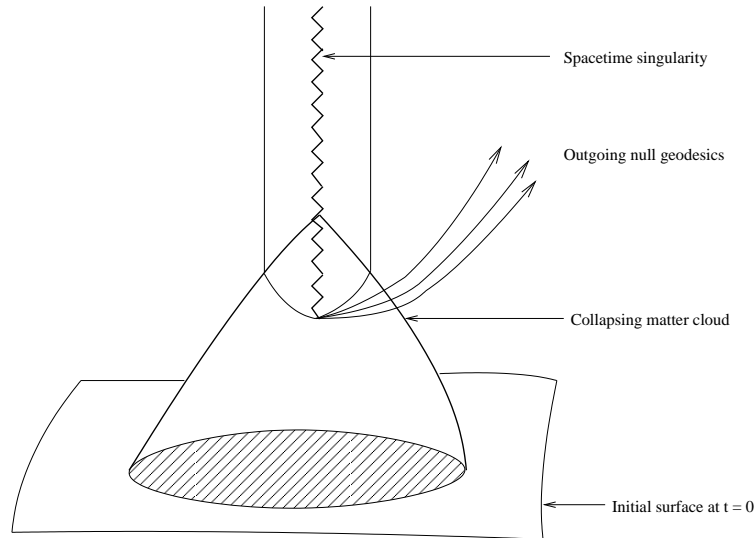


FIG. 2: When the density of collapsing cloud is higher at the center, light rays or particle trajectories can reach the external observer from the vicinity of the singularity.

implications. It appears likely from the current investigations that the astrophysical phenomena such as the Gamma Rays Bursts will have a strong connection to the physics and dynamics of gravitational collapse of massive stars.

The above discussion points to a wide variety of circumstances under which singularities develop in general relativistic cosmologies and in many gravitational collapse processes. Singularity theo-

rems imply the existence of vast classes of solutions to the Einstein equations that must contain spacetime singularities, as characterized by the conditions of these theorems, and of which the big bang singularity is one example. These theorems therefore imply that singularities must occur in Einstein's theory quite generically, that is, under rather general physically reasonable conditions on the underlying spacetime. Historically, this implication considerably strengthened our confidence in the big bang model which is used extensively in cosmology today.

While singularity theorems tell mainly on the existence part, what we really need is more information on the structure of the singularities in terms of their visibility or otherwise, curvature strengths and other such aspects. What is therefore called for is a detailed investigation of the dynamics of gravitational collapse within the framework of Einstein's theory.

In such a context, discussion of the gravitational collapse for specific models in general relativity can turn out to be of great help. One such model is that given by the Vaidya metric, which was originally developed by P. C. Vaidya in 1941 in the context of modelling a radiating star in general relativity. One can use this metric to study collapse of radiation shells within a Vaidya geometry, and it provided a great deal of information on the black hole and naked singularity formation in such collapse geometries [3], [4]).

#### IV. SINGULARITIES AND QUANTUM GRAVITY

Though general relativity deals with matter in the space-time as a purely classical entity that generates the curvature of space-time, we know that actually the matter and particles, and their interactions, obey and are governed by the laws of quantum theory. On very large scale in the universe, and at relatively lower matter densities, it may be possible to ignore the intrinsic quantum nature of the matter. In that case, general relativity provides us with fairly accurate predictions on the evolution of the universe.

The occurrence of singularities, however, offers us the regime where the matter densities, space-time curvatures, and gravity are all indeed extreme, and where the quantum gravity effects would be certainly important. As of today, we do not yet have a combined theory governing in a unified manner the forces operating within atom and at nuclear densities, and the force of gravity. Therefore a study of physical processes occurring in the vicinity of the space-time singularity would possibly offer a unique opportunity to study the physics of the gravity and the quantum together, and could possibly lead to a unified theory of all forces of nature. This is the cherished dream of the physicists which is to create a quantum theory of gravity. It is for this reason that the study

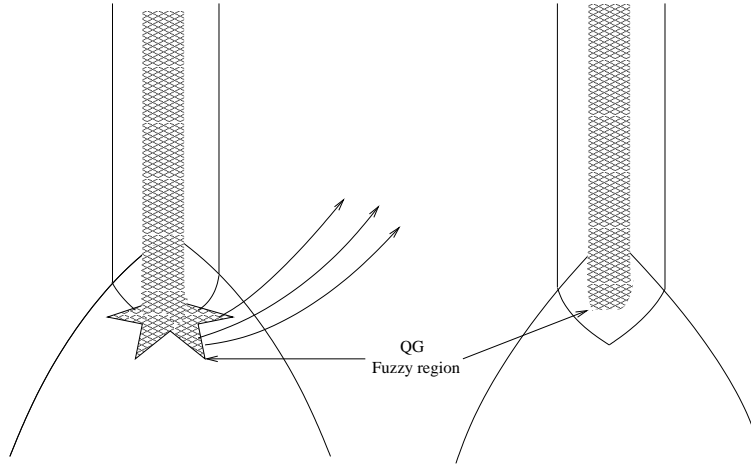


FIG. 3: Eventhough the singularity may resolve by the quantum gravity effects, the physical processes in the ultra-strong gravity regions may be seen by the observers far away in the universe. (*Figure courtesy: Ref 3 below of the PSJ book.*)

of space-time singularities occupies such a central place in fundamental physics today.

It is possible that such singularities represent the incompleteness of the theory of general relativity itself. Further, they may be resolved or avoided when quantum effects near the same are included in a more complete theory of quantum gravity. Nevertheless, there is a key point here. Even if the final singularity is dissolved by quantum gravity, what is really important is the inevitable occurrence of an ultra-strong gravity region, close and in the vicinity to the location of the classical singularity, either in cosmology or in dynamical processes involved in gravitational collapse. Such processes must affect the physics of the universe. An example of such a situation is the big bang singularity of cosmology. Even though such singularities may be possibly resolved through either quantum gravity effects, or due to features such as chaotic initial conditions, the effects of the super ultra-dense region of gravity that existed near the big bang epoch profoundly influence the physics and subsequent evolution of the universe. Similarly, we have in gravitational collapse of a massive star, the occurrence of singularities which are either visible to external observers or hidden behind the event horizon giving rise to a black hole. In either case again, the important issue is how the inevitably occurring super ultra-dense region would influence the physics outside.

We discussed here some aspects of space-time singularities. It is seen that in Einstein gravity they occur generically, whether covered within event horizons or as visible to external observers. If a future quantum theory of gravity resolves the final singularity of collapse or the initial one in cosmology, what is really interesting physically is the occurrence of regions of ultra-strong gravity

and space-time curvatures, that develop as the result of the dynamical gravitational processes.

The following physical picture then emerges. Dynamical gravitational processes proceed and evolve to create ultra-strong gravity regions in the universe. Once these form, strong curvature and quantum effects both come into their own in these regions. Quantum gravity then takes over and may resolve the final singularity. Particularly interesting is the case when the singularities of collapse are visible. In such a situation, quantum gravity effects, taking place in those ultra-strong gravity regions, will in principle be accessible and observable to external observers (see Fig.3). The consequences of such a scenario would be surely intriguing.

- 
- [1] R. M. Wald, *General Relativity*, University of Chicago Press, Chicago (1983).
  - [2] R. Geroch, ‘*The Structure of space-time*’, in *General Relativity- an Einstein Centenary Survey* (eds. S. W. Hawking and W. Israel), Cambridge University Press, Cambridge (1979).
  - [3] P. S. Joshi, ‘*Naked singularities*’, in *Scientific American*, February 2009.
  - [4] P. S. Joshi, *Gravitational Collapse and Spacetime Singularities*, Cambridge University Press, Cambridge (2007).